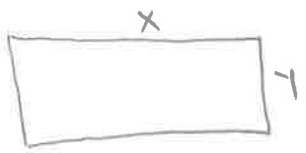
BC Calculus 4-4 optimization

Day 1 Pg 226 # 3, 6, 9, 11, 12, 14, 16

(3)



$P = 2x + 2y$ $A = xy$

$P(x) = 2x + 2\left(\frac{16}{x}\right)$ $16 = x \cdot y$

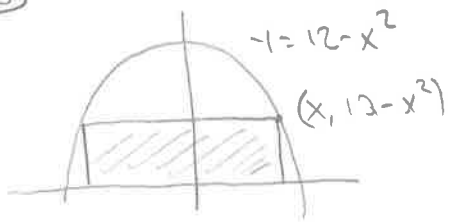
$y = \frac{16}{x}$

$\frac{dP}{dx} = 2 - \frac{32}{x^2}$

min Perimeter when

$x = 4$, $y = 4$, $P = 16$

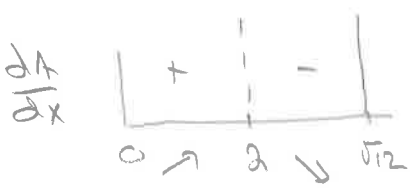
(6)



$A = 2x(12 - x^2)$

$A = 24x - 2x^3$

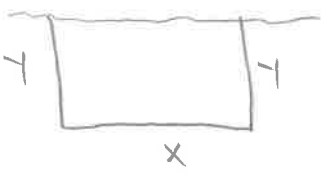
$\frac{dA}{dx} = 24 - 6x^2$



Max Area when $x = 2$

Dim: 4×8, Area = 32

(9)



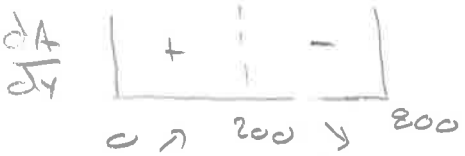
$800 = 2y + x$, $x = 800 - 2y$

$A = x \cdot y$

$A(y) = y(800 - 2y)$

$A(y) = 800y - 2y^2$

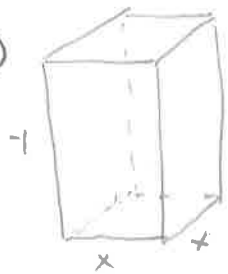
$\frac{dA}{dy} = 800 - 4y$



Max Area when $y = 200m$, $x = 400m$

Area: $80,000 m^2$

(11)



$V = x^2 y = 500$, $y = \frac{500}{x^2}$

$SA = x^2 + 4xy$

$SA = x^2 + 4x\left(\frac{500}{x^2}\right)$

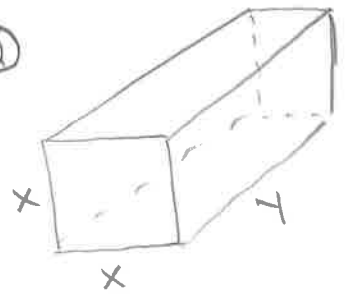
$A(x) = x^2 + \frac{2000}{x}$

$\frac{dA}{dx} = 2x - \frac{2000}{x^2}$

min surface area (weight) when

$x = 10$, $y = 5$

(12)



$V = x^2 y = 1125 \text{ ft}^3$

$y = \frac{1125}{x^2}$

$C = 5(x^2 + 4xy) + 10xy$

$C(x) = 5x^2 + \frac{22,500}{x} + \frac{11,250}{x}$

$= 5x^2 + \frac{33750}{x}$

$\frac{dC}{dx} = 10x - \frac{33750}{x^2}$

$\frac{dC}{dx}$ | − | + |

0 ↘ 15 ↗

min cost when

$x = 15$ ft, $y = 5$ ft

(14) $s(t) = -16t^2 + 96t + 112$

$v(t) = -32t + 96$

a) $v(0) = 96$ ft/sec

b) $v(t) = 0 = -32t + 96$

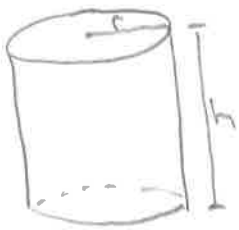
$t = 3$

$s(3) = 256$ ft

c) $s = 0$ when $t = 7$ (quad formula)

$v(7) = -128$ ft/sec

(16)



$V = \pi r^2 h$

$1000 = \pi r^2 h$

$h = \frac{1000}{\pi r^2}$

$SA = 2\pi r h + \pi r^2$

$A(r) = 2\pi r\left(\frac{1000}{\pi r^2}\right) + \pi r^2 = \pi r^2 + \frac{2000}{r}$

$\frac{dA}{dr} = 2\pi r - \frac{2000}{r^2} = 0$

$r^3 = \frac{1000}{\pi}$

$r = \frac{10}{\sqrt[3]{\pi}}$

$\frac{dA}{dr}$ | − | + |

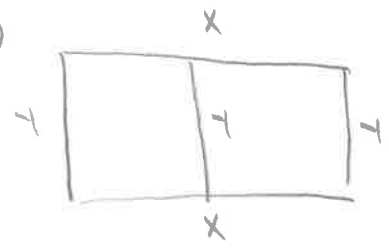
0 ↘ $\frac{10}{\sqrt[3]{\pi}}$ ↗

min SA when $r = \frac{10}{\sqrt[3]{\pi}}$

$h = \frac{1000}{\pi\left(\frac{10}{\sqrt[3]{\pi}}\right)^2} = \frac{10}{\sqrt[3]{\pi}}$

4-4 Day 2 Pg 226 # 10, 21, 25, 27, 32

(10)



$P = 2x + 3y$, $A = xy = 216\ m^2$

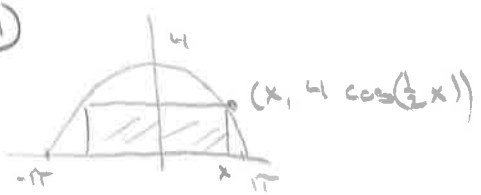
$P = 2\left(\frac{216}{y}\right) + 3y$

$\frac{dP}{dy} = 3 - \frac{432}{y^2}$

min Perimeter when

$y = 12m$, $x = 18m$ total: $72\ m$

(21)



$A(x) = 2x \cdot 4\cos\left(\frac{1}{2}x\right)$

$\frac{dA}{dx} = 8\cos\left(\frac{1}{2}x\right) - 4x \cdot \sin\left(\frac{1}{2}x\right)$

$= 0$ when $x = 1.720$

(from calculator)

Max Area $l = 3.44$, $w = 2.61$

Area = 8.98

(25) $C(x) = x^3 - 10x^2 + 30x$

Marginal cost: $C'(x) = 3x^2 - 20x + 30$

Average cost: $\frac{C(x)}{x} = x^2 - 10x + 30$

$3x^2 - 20x + 30 = x^2 - 10x + 30$

$2x^2 - 10x = 0$

$2x(x - 5) = 0$

$x = 0,\ x = 5$

$\frac{d^2}{dx^2}\left(\frac{C(x)}{x}\right) = 2 > 0$

$x = 5$ gives minimum

(5000 units)

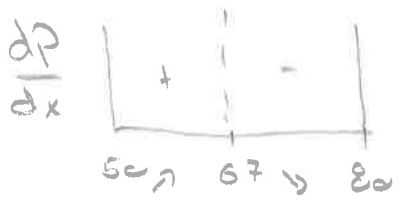
(27) $P(x) = r(x) - c(x)$

$P(x) = 200 \cdot 50 + x(200 - 2(x-50)) - (6000 + 32x)$

$= 10{,}000 + 300x - 2x^2 - 6000 - 32x$

$P(x) = -2x^2 + 268x + 4000$

$P'(x) = -4x + 268$



Max Profit : 67 People

(32) $V = \frac{1}{3}\pi r^2 h$

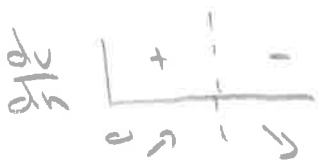
$h^2 + r^2 = (\sqrt{3})^2$

$r^2 = 3 - h^2$

$V = \frac{1}{3}\pi(3 - h^2) \cdot h$

$V(h) = \pi h - \frac{1}{3}\pi h^3$

$\frac{dV}{dh} = \pi - \pi h^2 = \pi(1 - h^2)$



Max volume when

$h = 1, \; r = \sqrt{2}$

$V = \frac{2\pi}{3}\, m^3$